GCSE Maths – Geometry and Measures

Congruent Triangles

Notes

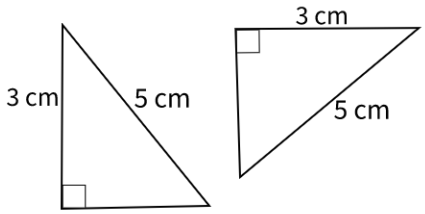
WORKSHEET

Congruent Triangles

Two triangles are **congruent** if the size of **all three pairs of angles and sides** are **the same.**

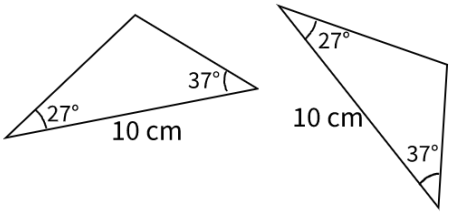
Conditions for congruency

To show that two triangles are congruent, you must **prove** one of the following four conditions are true:



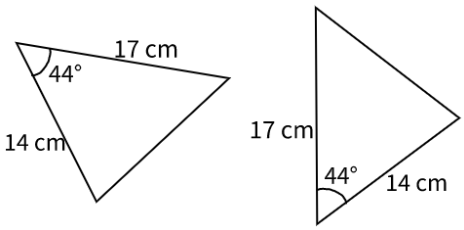
RHS (right-angle, hypotenuse, side)

- Both triangles have a right-angle.
- Both triangles have the same hypotenuse length.
- Have one other pair of sides which are equal



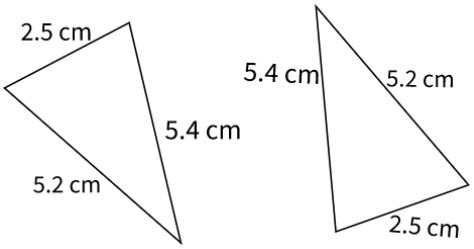
ASA (angle, side, angle)

- Two pairs of angles are equal.
- The side between the angles are the same.



SAS (side, angle, side)

- Two pairs of sides are equal.
- The angle between the pair of equal sides is the same.

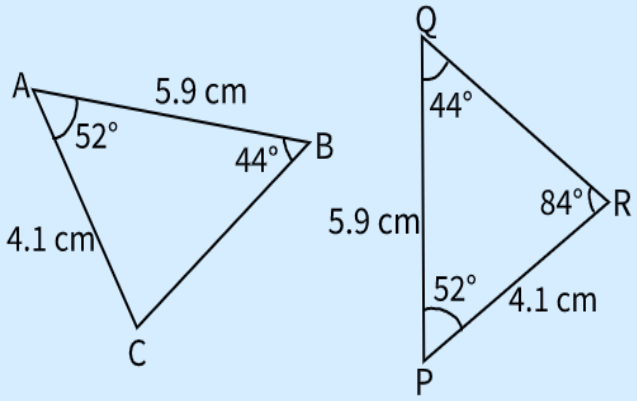


SSS (side, side, side)

- All three pairs of sides are equal.

Often in questions, the pairs of triangles will have **different orientations**, but this has no effect on whether they are congruent or not.

Example: Are triangles ABC and PQR congruent? Explain your answer.



1. **Identify similarities** between triangles ABC and PQR.

   Similarities in angles:

   $$\angle CAB = 52°$$
   $$\angle QPR = 52°$$

   So, we see that

   $$\angle CAB = \angle QPR$$

   Also,

   $$\angle ABC = 44°$$
   $$\angle PQR = 44°$$

   So, we see that

   $$\angle ABC = \angle PQR$$

   Similarities in sides:

   $$AB = 5.9\text{ cm}$$
   $$PQ = 5.9\text{ cm}$$

   So, we see that

   $$AB = PQ.$$

2. Use the information to decide if you can use any of the four conditions to prove congruency of the triangles.

   We have deduced that the triangles have two pairs of angles which are equal:

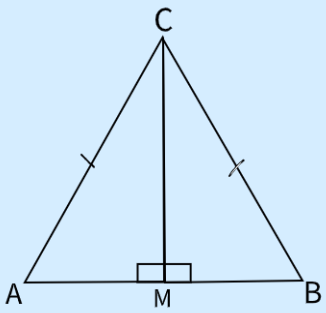
   $$\angle CAB = \angle QPR$$
   $$\angle ABC = \angle PQR$$

   We have shown the corresponding side between the angles is the same:

   $$AB = PQ$$

   Therefore, the condition ASA has been met since we have two pairs of matching angles and the side between them is the same length. Therefore, the triangles are congruent.

Example: In the diagram below, point M is the midpoint of AB. Prove that ΔAMC and ΔBMC are congruent.



1. **Identify similarities** between triangles **AMC** and **BMC**.

   The dashes on the lines indicate these lines are the same length. So,

   $$AC = CB.$$

   Since M is the midpoint of AB, then

   $$AM = BM.$$

   Both triangles have the line CM in common, so this is another side of equal length.

   Both triangles have a right angle.

2. Use the information to decide if you can prove any of the four conditions for congruency.

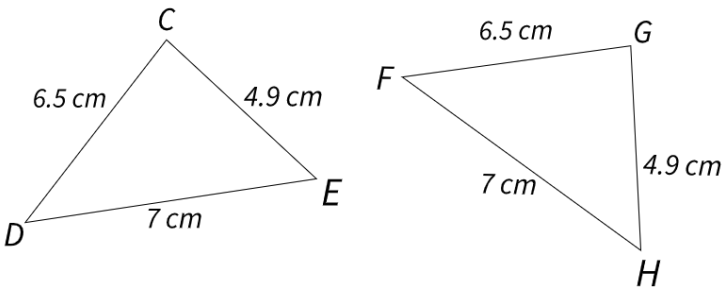
   We have deduced that the triangles have a side and hypotenuse which are equal, and both have a right angle.

   Therefore, the condition RHS has been met. This means we can conclude that the triangles are congruent.

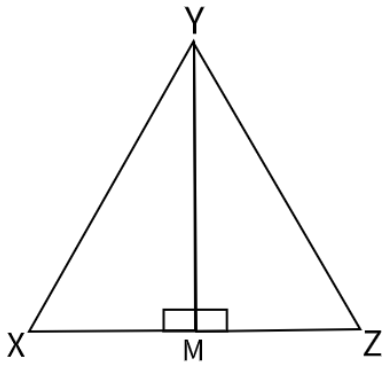
   Alternatively, we can prove the condition SSS has been met since we have identified three equal sides.

Congruent Triangles – Practice Questions

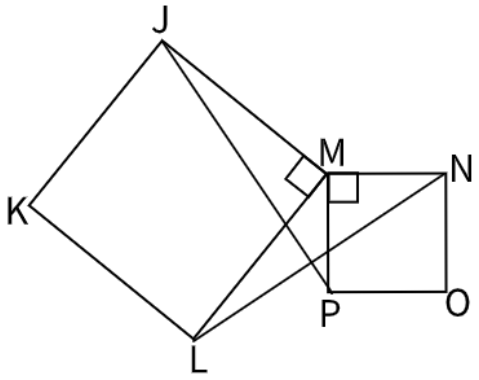
1. Are these triangles congruent? Explain your answer.



2. Are these triangles congruent? Explain your answer.



3. Given that $JKLM$ and $MNOP$ are squares, are the triangles JPM and LMN congruent?



Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.